Age-Optimal Status Updating with an Energy Harvesting Sender

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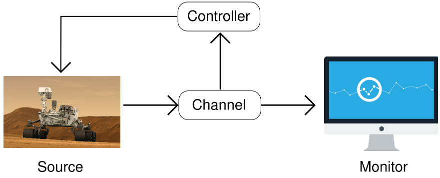
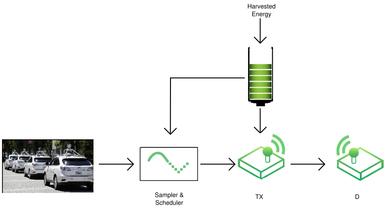
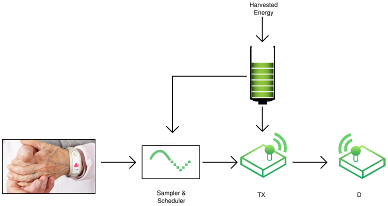
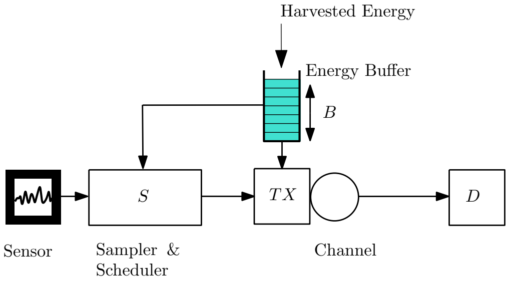
Motivation for the Problem







Problem Formulation



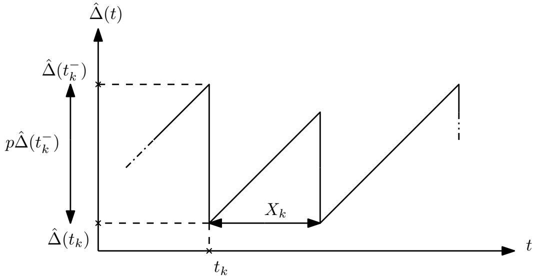
- Can send one update per energy packet
- Packet erasure channel, no feedback
- Poisson energy packet arrivals (rate μ_H)
- Battery limit B

Related Work

- T. Bacinoglu, E.T. Ceran & E. Uysal-B., ”Age of information under energy replenishment constraints”, ITA 2015
  - Energy harvesting (”energy causality") constraints, deterministic delay
- R. Yates, "Lazy is Timely: Status updates by an energy harvesting source,” ISIT 2015
  - Average energy constraint, random delay
- R. Yates, E. Najm, E. Soljanin, J. Zhong,"Timely updates over an Erasure Channel”, 2017
- J. Yang, ITA 2017

Problem Formulation

- Status age $\Delta(t) = t - u(t)$
- Age estimate at S:
  $\hat{\Delta}(t) = \mathbb{E}[\Delta(t) \mid \underbrace{\{t_k : k \geq 1, t_k \leq t\}}_{\text{previous update instants}}]$



Problem Formulation

- Objective:

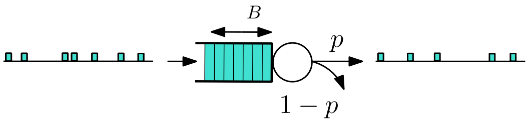
  Minimize $\bar{\Delta} = \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T \Delta(t)dt\right]$

- Equivalently:

  Minimize $\limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T \hat{\Delta}(t)dt\right]$

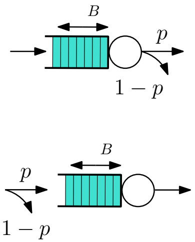
- CT MDP w/ Continuous State-Space
- State: $(E(t), \hat{\Delta}(t))$

Alternative Motivation for the Problem



- No control: *Zero Wait*
  - Only work-conserving policy
  - Poisson output
- Find non work-conserving policy for service regulation
- Age-controlled (second moment regulation)

Lemma 2



Erase then Regulate vs. Regulate then Erase

$$\bar{\Delta}_B^{\text{opt}}(p\mu_H, 1) \le \bar{\Delta}_B^{\text{opt}}(\mu_H, p)$$

$p = 1$ Case

$(\underbrace{E(t_k)}_{\text{Energy state}}, \underbrace{t_k}_{\text{Jump times}})$: **Markov Renewal Process.**

$A(t) := E(t_k)$ for $t_{n-1} \leq t < t_n$: **Semi-Markov Process.**

Special case: $(B = 1)$
$E(t_k)$ reset to 0 after each transmission $\Rightarrow X_k = t_k - t_{k-1}$
Renewal Process.

Lower bound for any B, p

- Theorem 2: $\bar{\Delta}^{\text{opt}} \geq \frac{1}{2p\mu_H}$
- Proof sketch:
  - Express average age for $p = 1$, use convexity
  - Use Lemma 2 for general p
- Tight for $p = 1$, $B \rightarrow \infty$
  - Achieved by deterministic policy: transmit at $1/\mu_H + \epsilon$ intervals) for any $\epsilon > 0$

Upper and lower bounds for $B = \infty$, any p

- Deterministic policy asymptotically achieves $\bar{\Delta}^{\text{det}} = [1/2 + (1-p)/p](1/\mu_H)$
- By Theorem 2, $\bar{\Delta}^{\text{opt}} \geq \frac{1}{2p\mu_H}$
- So $\frac{U}{L} \leq \frac{\bar{\Delta}^{\text{det}}}{\bar{\Delta}^{\text{opt}}} = 2 - p$
- $\Rightarrow$ Tight for $p = 1$

Threshold Policies

- Given a set of thresholds $[\tau_1, \tau_2, ..., \tau_B]$
- For $E(t) = m$, update if $\hat{\Delta}(t) \geq \tau_m$
- $\tau_0 = \infty$
- Optimality of threshold policy shown at ITA 2015

Zero-wait is a threshold policy

- $\tau_m = 0$ for all $1 \leq m \leq B$
- Provides upperbound for threshold policies, any B and p:

$$\min_{\{\tau_1,\tau_2,\dots,\tau_B\}\in[0,+\infty)^B} \bar{\Delta}^{\text{OPT}} \leq \bar{\Delta}^{\text{ZW}} = \frac{1}{p\mu_H}$$

Average age for threshold policies

Consider $B = 1$

- For $p = 1$, as X_k iid use renewal reward argument

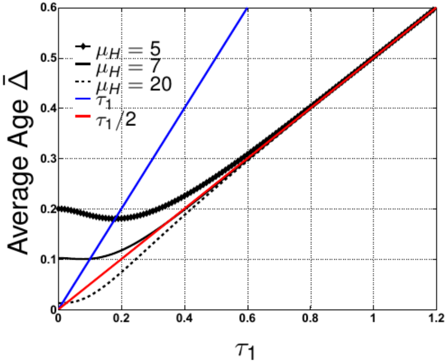
$$\bar{\Delta} = \frac{1}{2} \frac{\tau_1^2 + (\frac{2}{\mu_H^2} + \frac{2}{\mu_H}\tau_1)e^{-\mu_H \tau_1}}{\tau_1 + \frac{1}{\mu_H} e^{-\mu_H \tau_1}}$$

- Optimization over τ_1 gives:
  - $\tau_1^* = \frac{2W(\frac{1}{\sqrt{2}})}{\mu_H}$ ($W(\cdot)$ being the Lambert-W function)
  - $\bar{\Delta}_1^{\text{opt}}(\mu_H, 1) = \tau_1^* = \frac{2W(\frac{1}{\sqrt{2}})}{\mu_H} \approx \frac{0.901}{\mu_H}$
- By Lemma 2, $\bar{\Delta}_1^{\text{opt}}(\mu_H, p) \geq \frac{2W(\frac{1}{\sqrt{2}})}{p\mu_H}$

Corollary

- For $B = 1, p = 1$:
  mean inter-update duration $\approx 1.307 \times \frac{1}{\mu_H}$
- Echoes ”Lazy is Timely" result of Yates

Average age for $p = 1$



- $\tau^* = \bar{\Delta}_1^{\text{opt}}$
- Asymptotically Δ grows like $\tau/2$

Average age for $p = 1$

Consider $B = 2$, $\tau_2 \leq \tau_1$:

$$\Pr(X_k \leq x \mid E_{k-1} = 0) = \begin{cases} 0 & x < \tau_2 \\ \Pr(W_2 \leq x) & \tau_2 \leq x < \tau_1 \\ \Pr(W_1 \leq x) & \tau_1 \leq x \end{cases}$$

$$\Pr(X_k \leq x \mid E_{k-1} = 1) = \begin{cases} 0 & x < \tau_2 \\ \Pr(W_1 \leq x) & \tau_2 \leq x < \tau_1 \\ 1 & \tau_1 \leq x \end{cases}$$

where $W_m \sim Erlang(m, \mu_H)$ and $E_k = E(t_k)$.
For $j = 0, 1$:

$$\Pr(E_k = 0 \mid E_{k-1} = j) = \Pr(W_{2-j} > \tau_1)$$

$$\Pr(E_k = 1 \mid E_{k-1} = j) = \Pr(W_{2-j} \leq \tau_1)$$

Average age for $p = 1$

Special case: $(B = 2)$

$$\bar{\Delta} = \frac{\sum_{j=0}^{1} \mathbb{E}[X_k^2 \mid E_{k-1} = j]\pi_j}{2\sum_{j=0}^{1} \mathbb{E}[X_k \mid E_{k-1} = j]\pi_j}$$

where π_j is the steady-state probability of state j:

$$\pi_j = \sum_{i=0}^{1} \Pr(E_k = j \mid E_{k-1} = i)\pi_i$$