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Optimal Sampling and Scheduling for Remote Fusion Estimation of Correlated Wiener Processes

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Outline

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I Motivation

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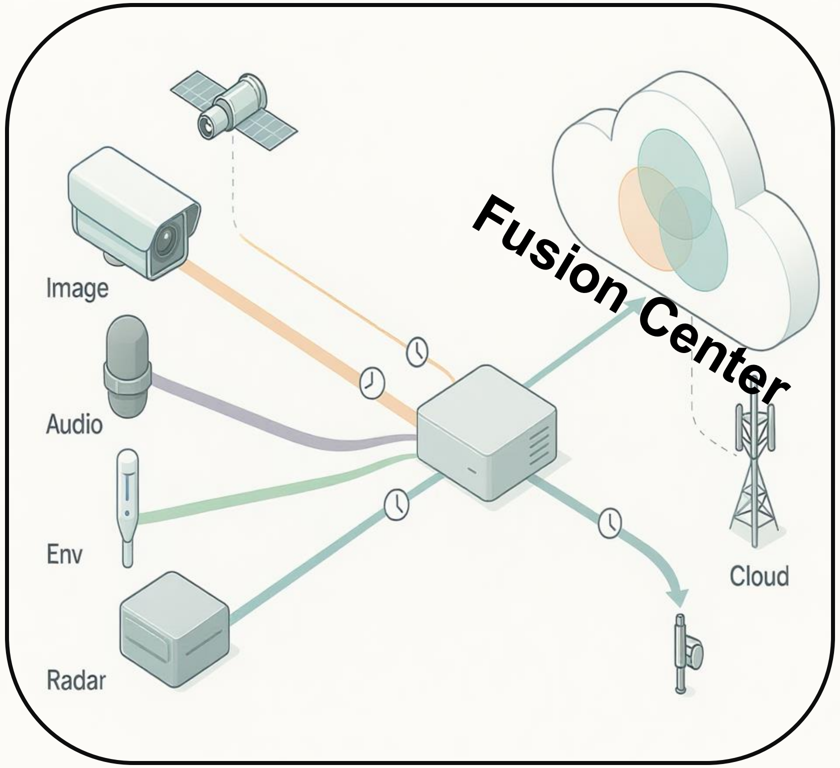
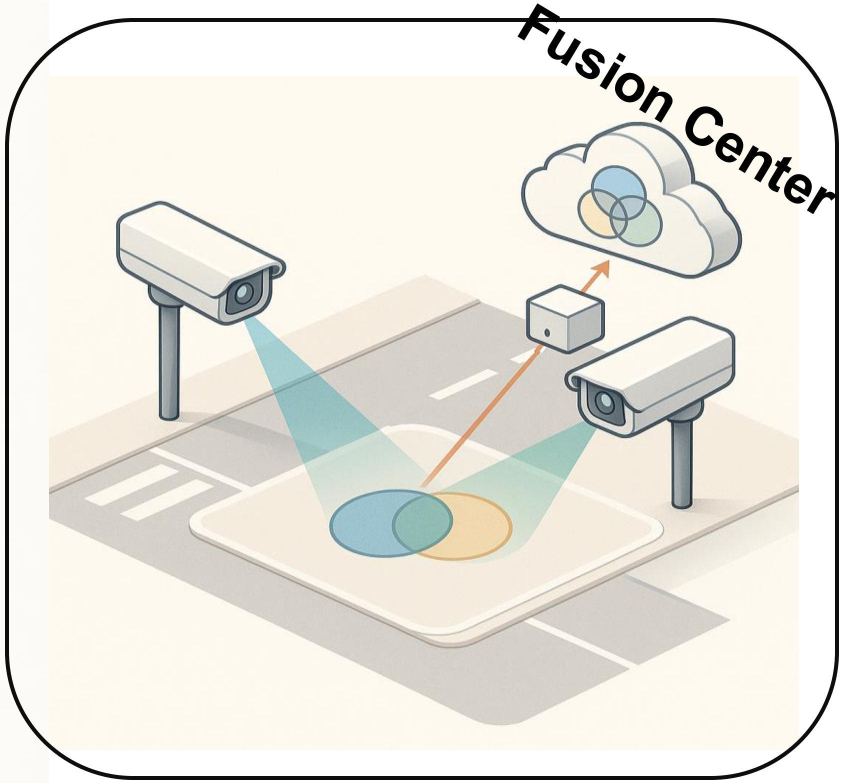
Motivation: Goal-Oriented Communications

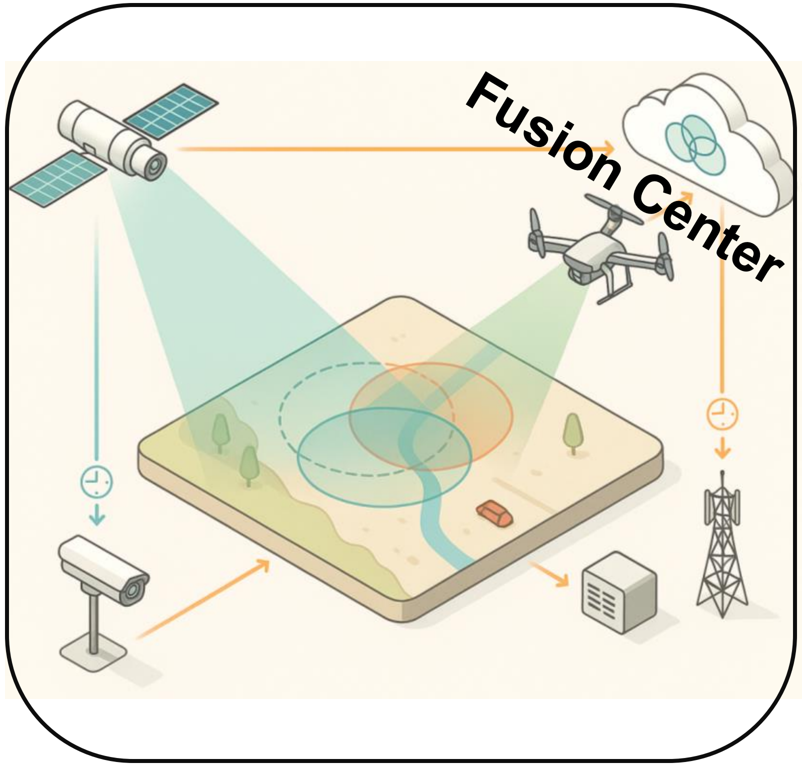
- **Scenario: Multiple Correlated Sources**



Correlated Multi-Modal Data Comming From Heterogeneous Sensors

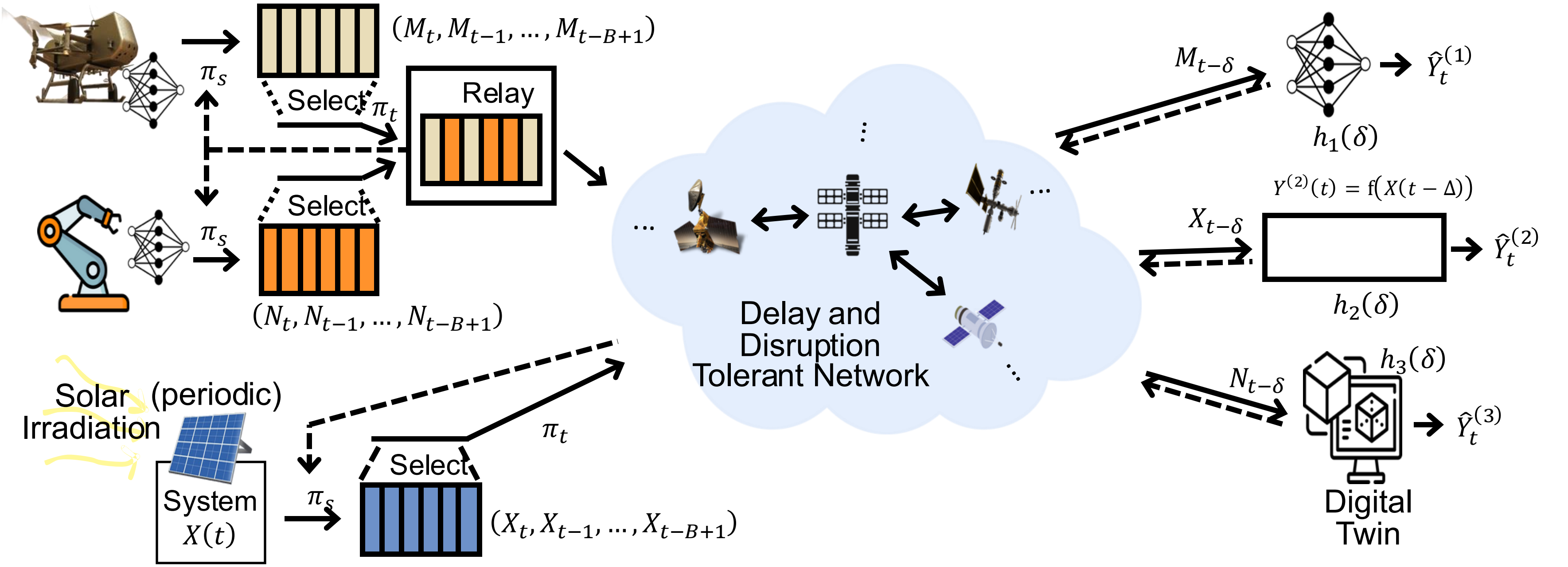


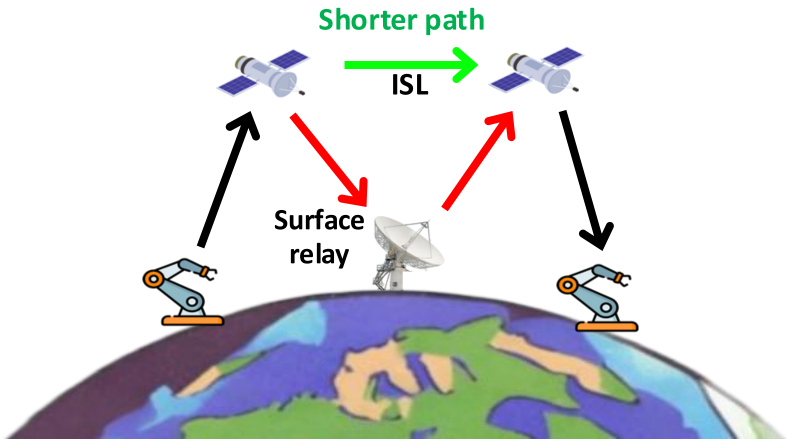
Distributed Sensors Observing an Overlapping Area



Cross-Platform Situational Awareness (Satellite–UAV–Ground)

Motivation: Goal-Oriented Communication





Variable delay due to links being unavailable

Motivation: Goal-Oriented Communication

Open Challenge: How to sample and transmit, to optimize **goal-oriented application-layer performance** in the presence of **network delay**

Prior insights (Single Source):

- **Wiener process estimation[2]**: **threshold policy for sampling f(change, age)**
- **Single-Source OU process estimation[3]**: **MSE monotonic in age**
- **General inference**: **non-linear, non-monotonic performance degradation in AoI**

Open problem: Multiple sources (correlated, delayed):

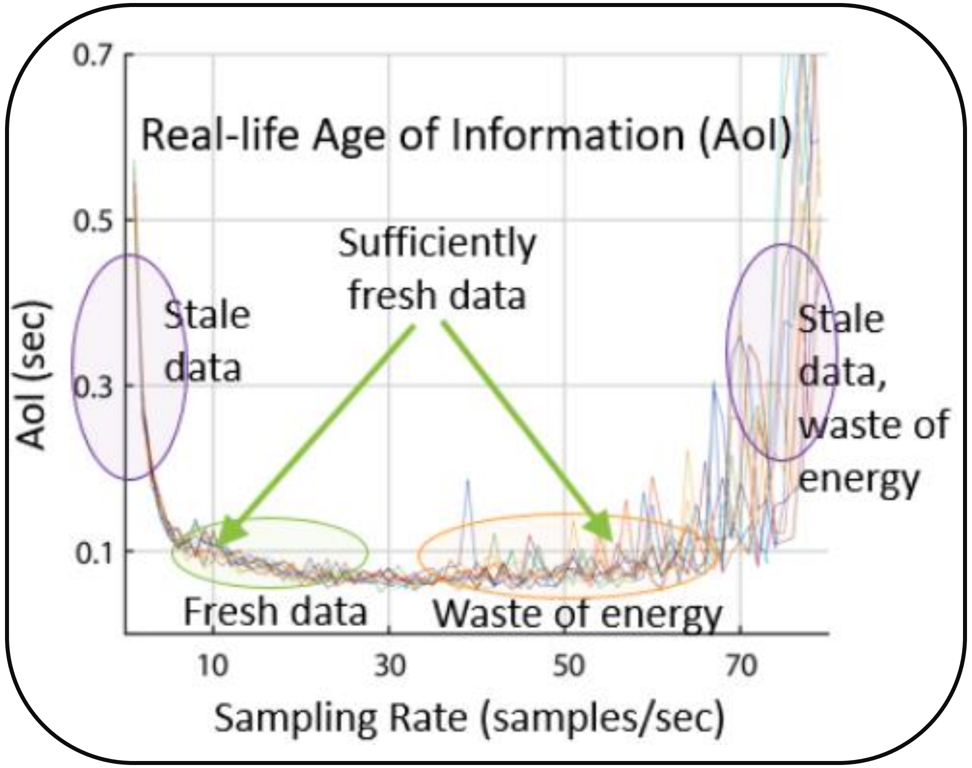
- Most studies assume **independent** multiple sources
- Heuristic metrics for correlated sources: **Age of Correlated Information**, etc.
- **Our formulation**: optimal pull-based sampling for remote **joint estimation**

[2] Y Sun, Y Polyanskiy, E Uysal. Sampling of the Wiener Process for Remote Estimation over a Channel with Random Delay. IEEE Transactions on Information Theory, 2019

[3] TZ Ornee, Y Sun. Sampling and remote estimation for the Ornstein-Uhlenbeck process through queues: Age of information and beyond. ACM/IEEE Transaction on Networking, 2021

Motivation: Age and Beyond

- **Insights from Age of Information (AoI) research**



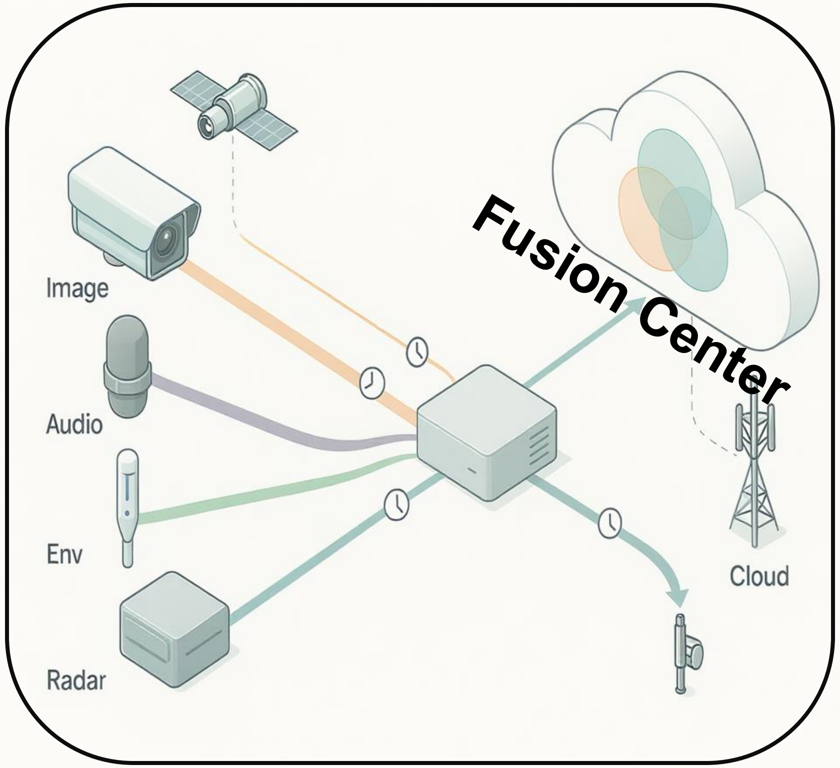
- AoI captures the freshness of information:
  Sparse Update→ Stale;
  Frequent Update→ Waste of Energy+Stale.
- AoI as a key metric whenever **fresher data is more valuable**.
- Age-optimal policy neither transmits at the highest possible rate nor requires the smallest end-to-end delay [1] **(Waiting before transmitting may be efficient).**

Open Challenge: **Impact of AoI** on **goal-oriented application-layer performance** remains not fully understood.

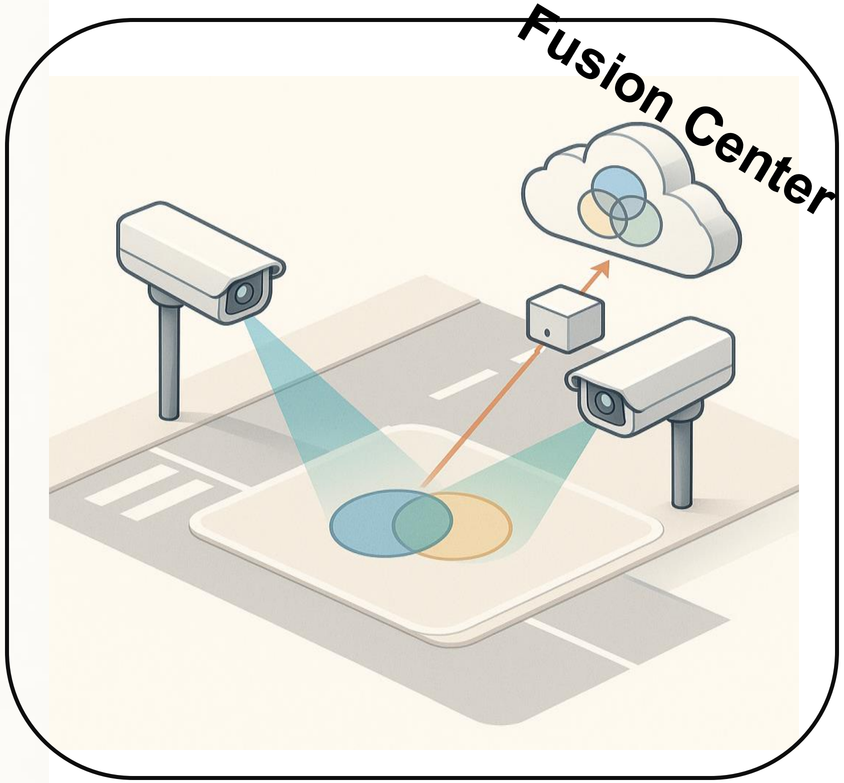
[1] Y Sun, E Uysal, RD Yates, CE Koksal, NB Shroff. Update or Wait: How to Keep Your Data Fresh. IEEE Transaction on Information Theory, 2017.

Motivation: Goal-Oriented Communications

- **Scenario: Multiple Correlated Sources**



Correlated Multi-Modal Data Comming From Heterogeneous Sensors



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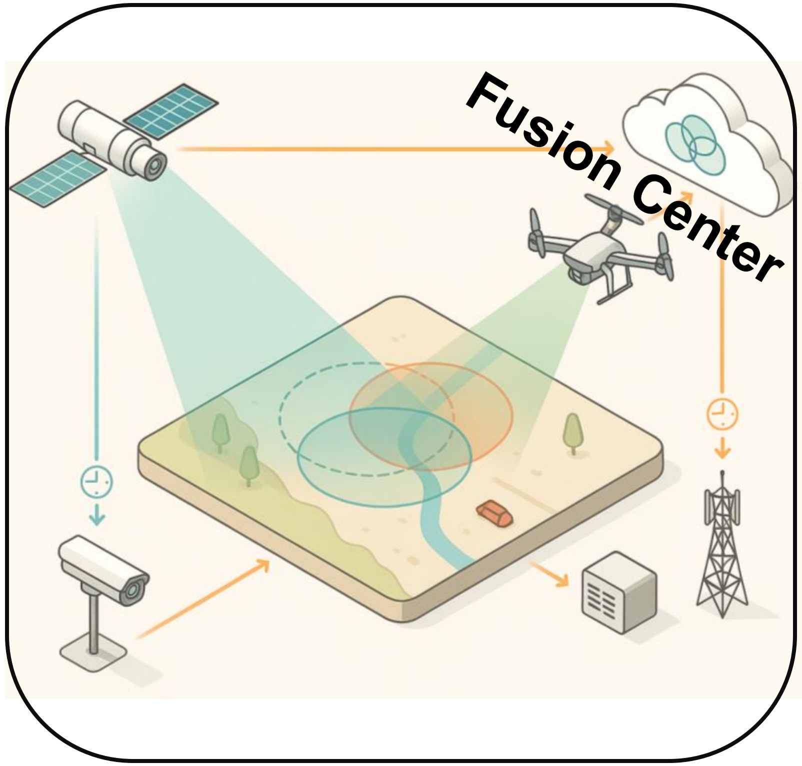

Cross-Platform Situational Awareness (Satellite–UAV–Ground)

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II Problem Formulation

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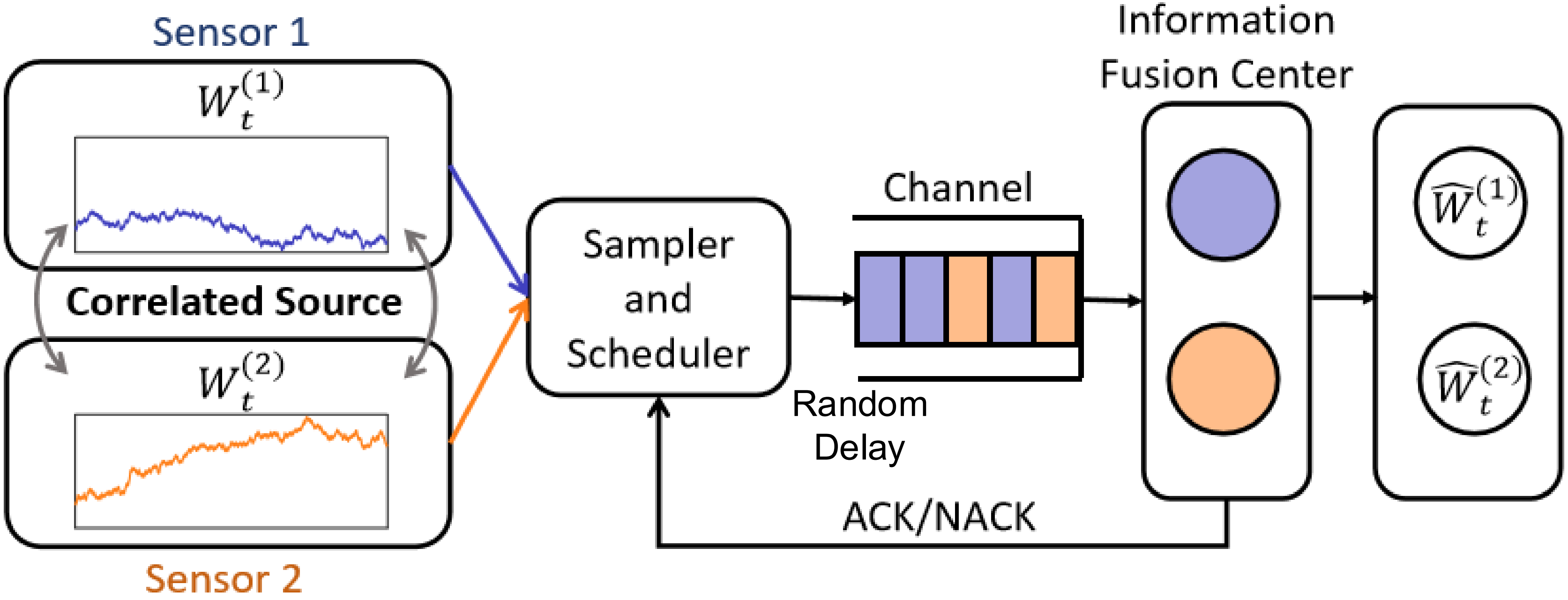
Motivation: MSE-Oriented Remote Information Fusion

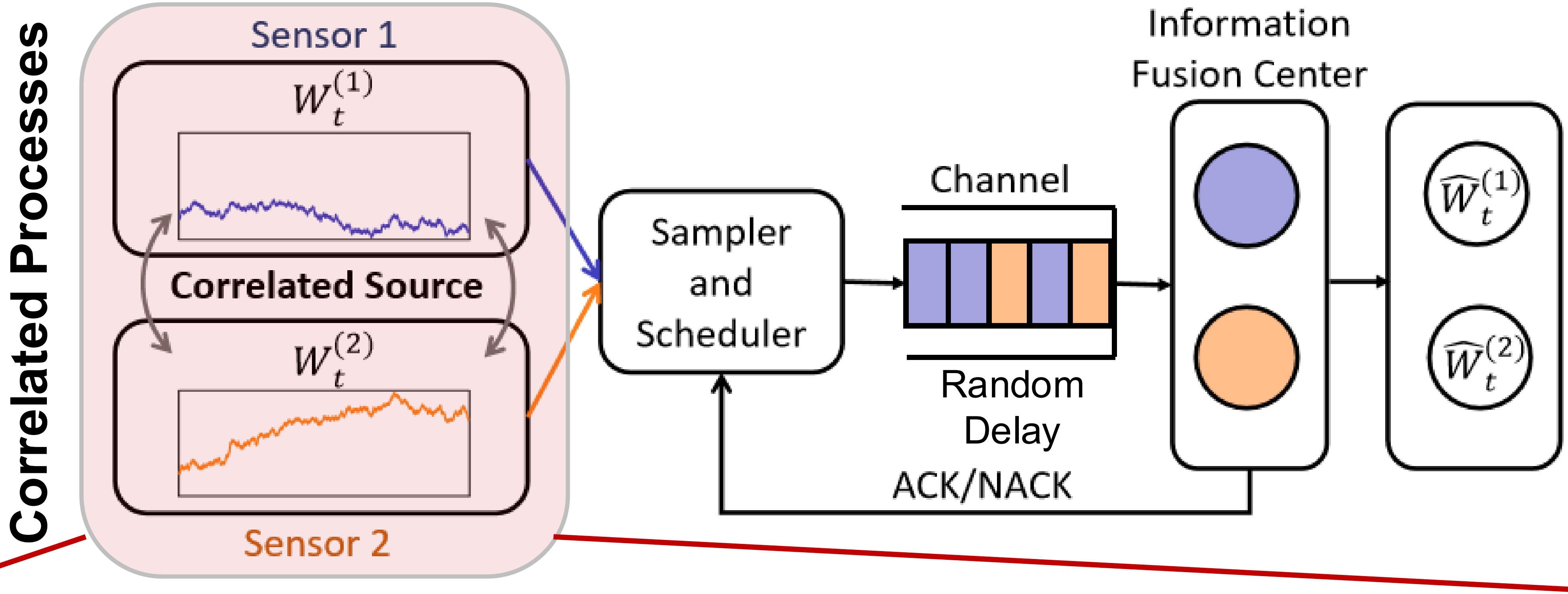
- Information Fusion and Estimation



Challenge: Naïve aggregation of **delayed data from correlated sources** may lead to redundant or misleading information.

Question: How to sample to optimize the staleness level.

System Model: Multiple Correlated Sources



Multiple Sources: Correlated Wiener Processes

Source Dynamics:

The two observed sources $W_t^{(1)}$ and $W_t^{(2)}$ are **Correlated Wiener process**:

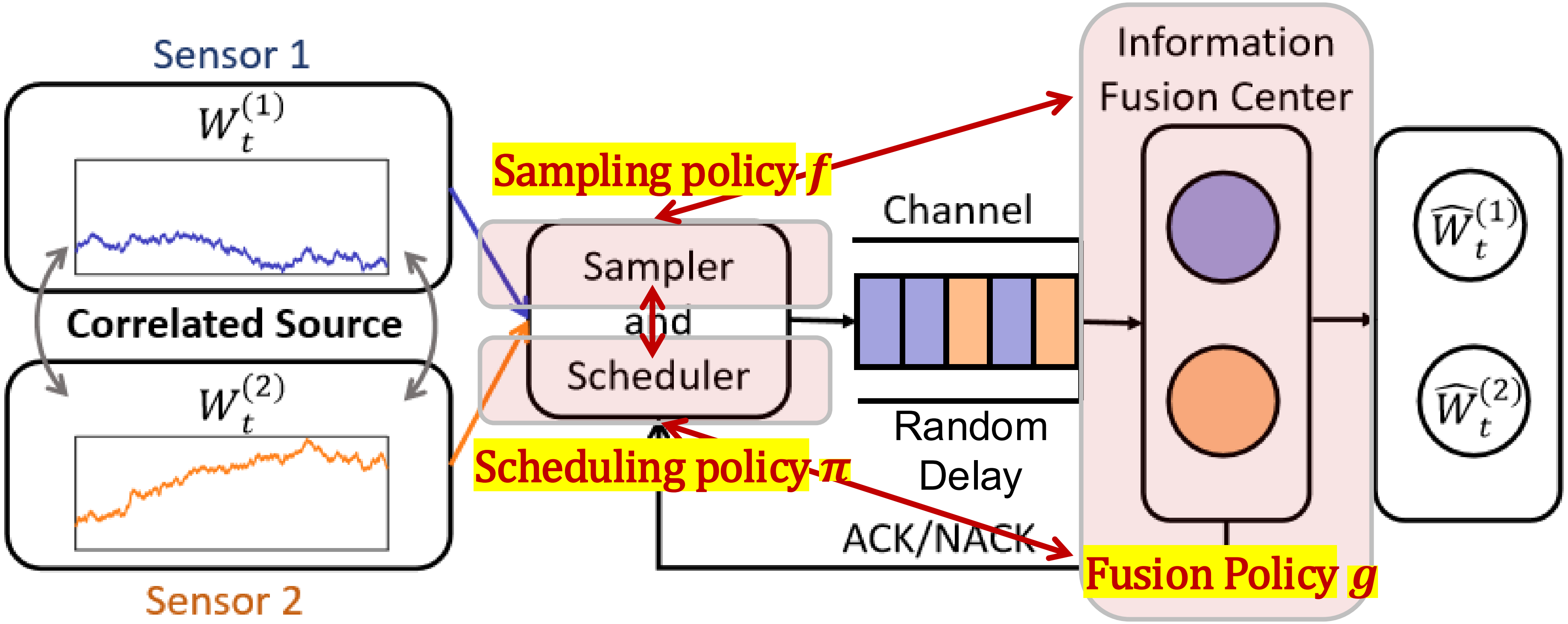
$$dW_t^{(1)} = dB_t^{(1)}$$

$$dW_t^{(2)} = \rho dB_t^{(1)} + \sqrt{(1-\rho^2)}dB_t^{(2)}$$

Mutually Independent Wiener Process

ρ correlation coefficient; if $\rho = 1$, $W_t^{(2)} = W_t^{(1)}$; if $\rho = 0$, $W_t^{(2)}$ is independent of $W_t^{(1)}$

System Model: Objective



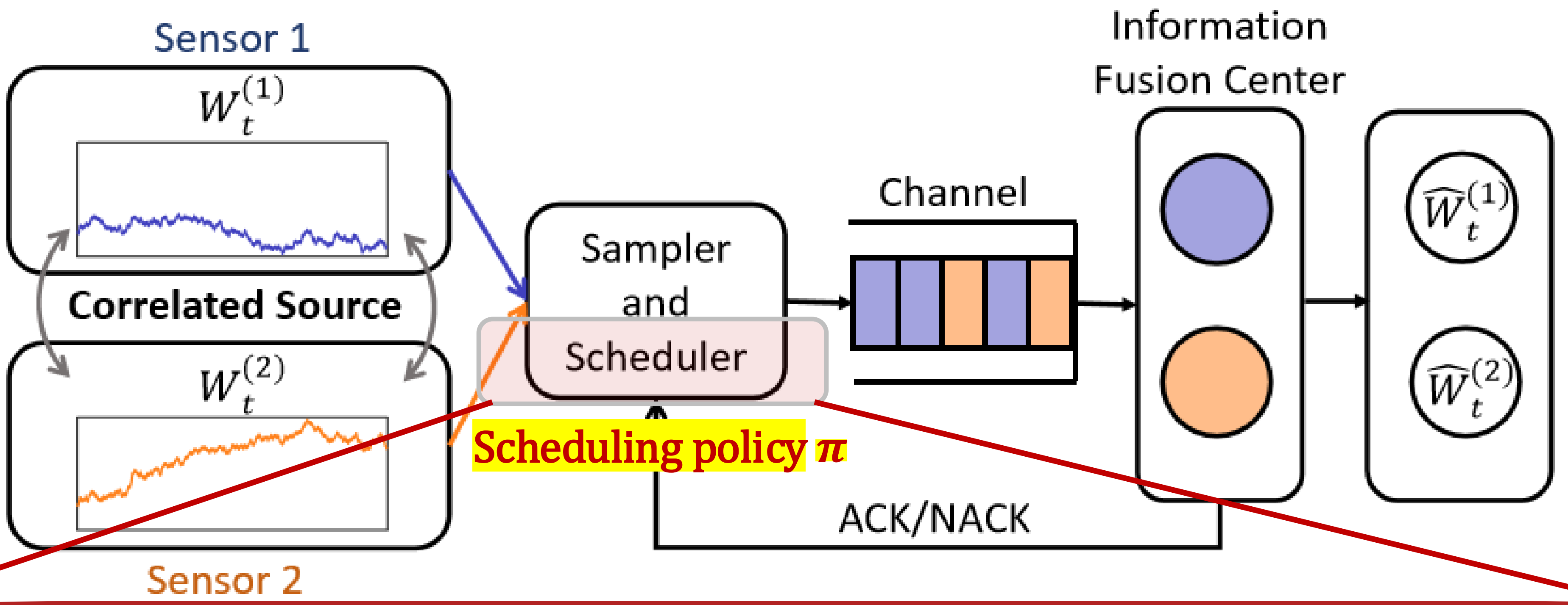
Co-Design Sampler, Scheduler, and Remote Information Fusion

Objective:

- Find out the optimal sampling, scheduling, remote estimation to minimize the long-term average **MSE**

$$\inf_{g,\pi,f} \limsup_{T\to\infty} \frac{1}{T} \mathbb{E}\left[\int_0^T (W_t^{(1)} - \widehat{W}_t^{(1)})^2 + (W_t^{(2)} - \widehat{W}_t^{(2)})^2 dt\right]$$

System Model: Scheduling Policy Definition



Scheduler Design (Which Source to Transmit?)

Scheduling Policy π

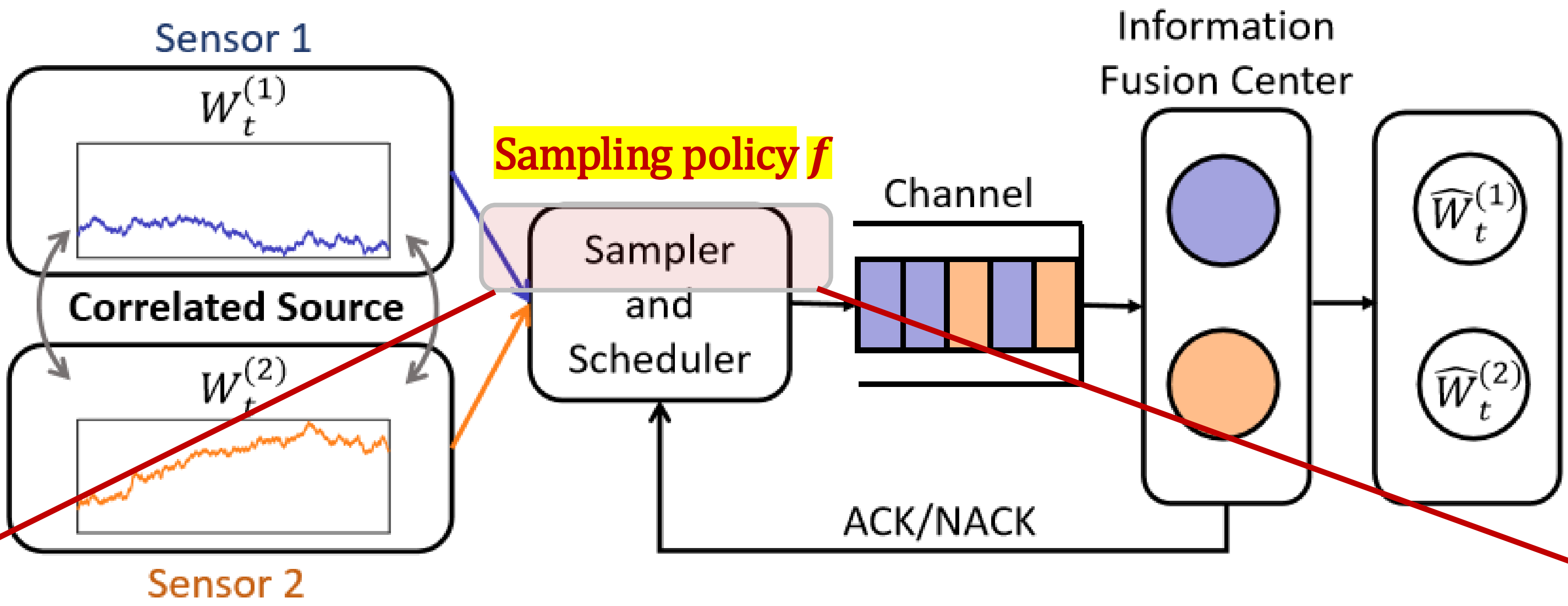
A scheduler is a **policy function** that determines **which source is selected for transmission**.

$$a_i = \pi(\mathcal{I}_{D_{i-1}}), \qquad \forall\, i \in \mathbb{N}$$

Here $a_i \in \{1,2\}$ is the scheduling action; $\mathcal{I}_{D_{i-1}}$ is the history before the time D_{i-1}

$$\mathcal{I}_t = \{a_j, S_j, W_{S_j}^{(a_j)}, D_j | D_j \le t\}$$

System Model: Sampling Policy Definition



Sampler Design (When to Transmit?)

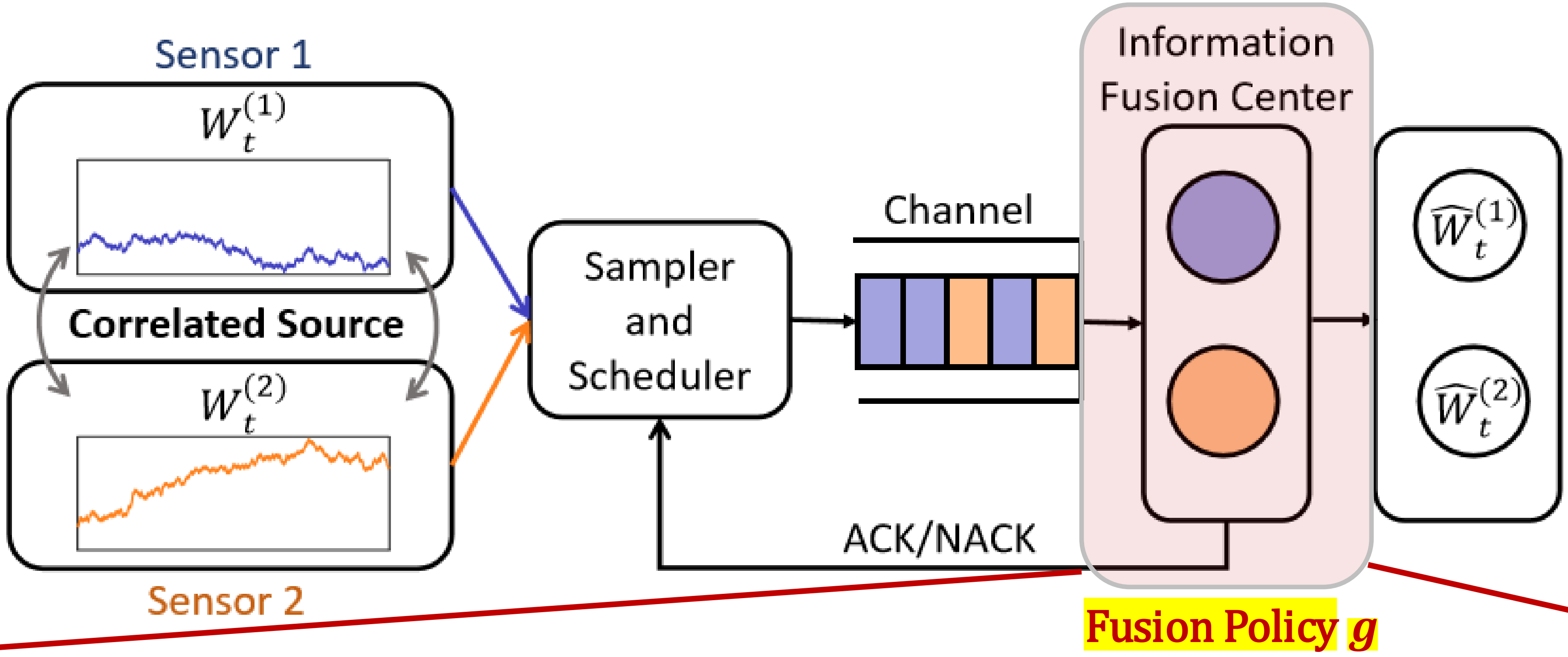
Sampling Policy f

A sampler is a policy function that determines **when each sensor is sampled and transmitted.**

$$S_i = f(\mathcal{I}_{D_{i-1}}), \qquad \forall\, i \in \mathbb{N}$$

Here $S_i \in \mathbb{R}^+$ is the sampling time of the i-th transmitted packet;

System Model: Fusion Policy Definition



Remote Information Fusion

Fusion policy g

A function that determines the real-time estimate $(\widehat{W}_t^{(1)}, \widehat{W}_t^{(2)})$:

$$(\widehat{W}_t^{(1)}, \widehat{W}_t^{(2)}) = g(\mathcal{I}_t), \qquad \forall\, t \in \mathbb{R}^+$$

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III Main Results

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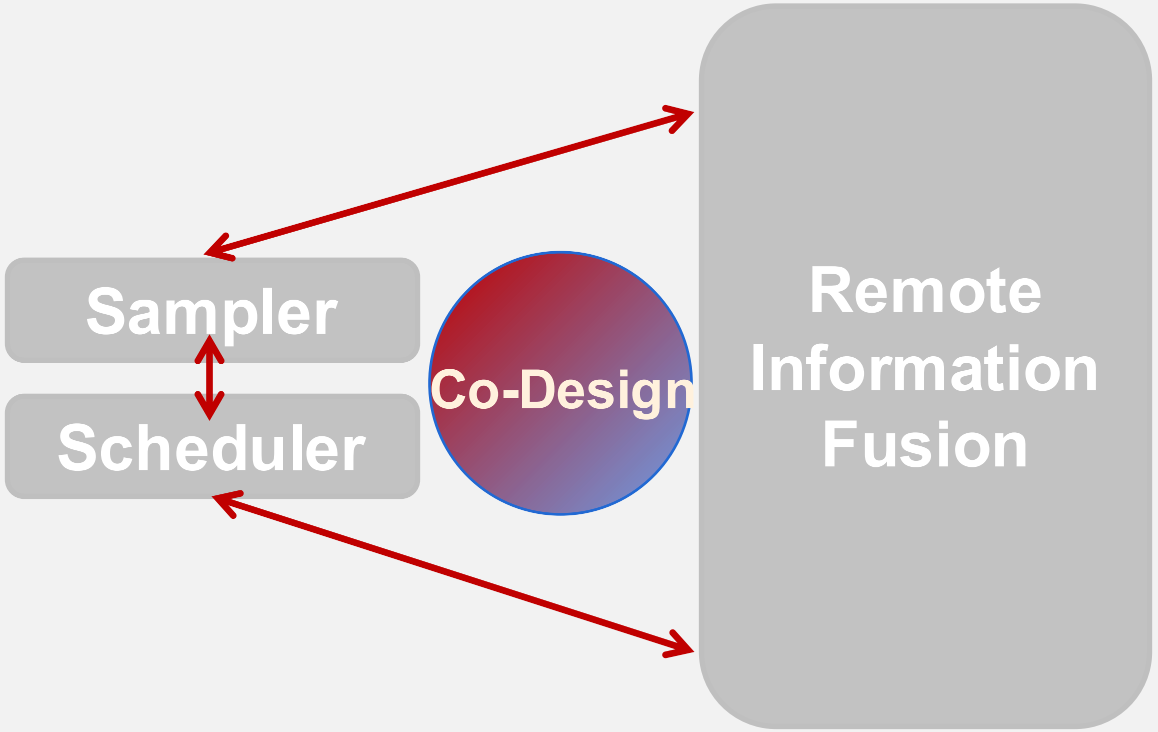
Main Results: Separation Principle

Coupled Policies



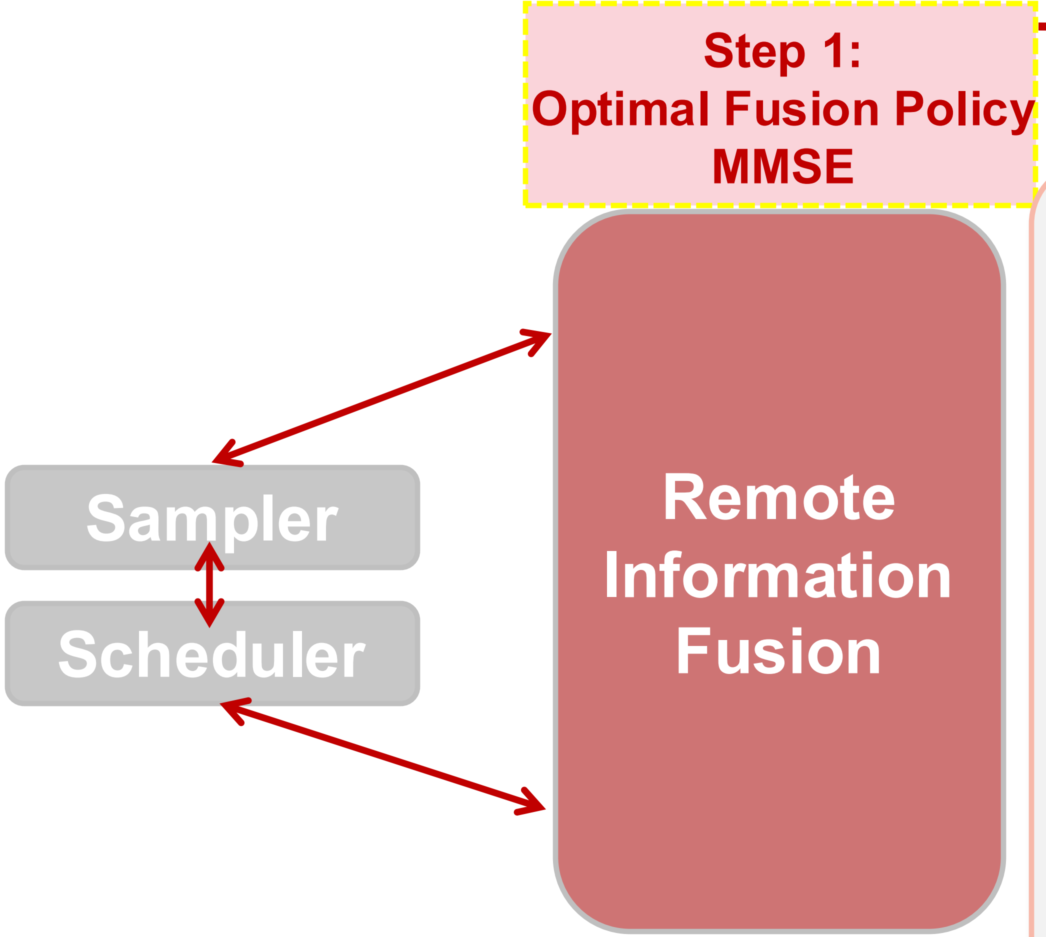
- **Challenge**
  - These three policies are mutually coupled.
  - Challenging for explicit optimal policies.

Our work achieves explicit solutions

Separation Principles

- **Optimal Remote Information Fusion**
  - Given any sampling and scheduling policy pair, the **MMSE fusion policy is optimal**.
- **Optimal Scheduling Policy**
  - Given any sampling policy and MMSE remote fusion, the optimal scheduling policy is the **Maximum Age First (MAF) scheduling policy**.
- **Optimal Sampling Policy**
  - Given the remote MMSE fusion policy and MAF scheduling policy, the optimal scheduling policy is the **threshold-based Age-optimal sampling policy**.

Main Results: Optimal Remote Information Fusion



Decoupled Fusion Policy

Optimal Fusion Policy given any π and f (Theorem 1):

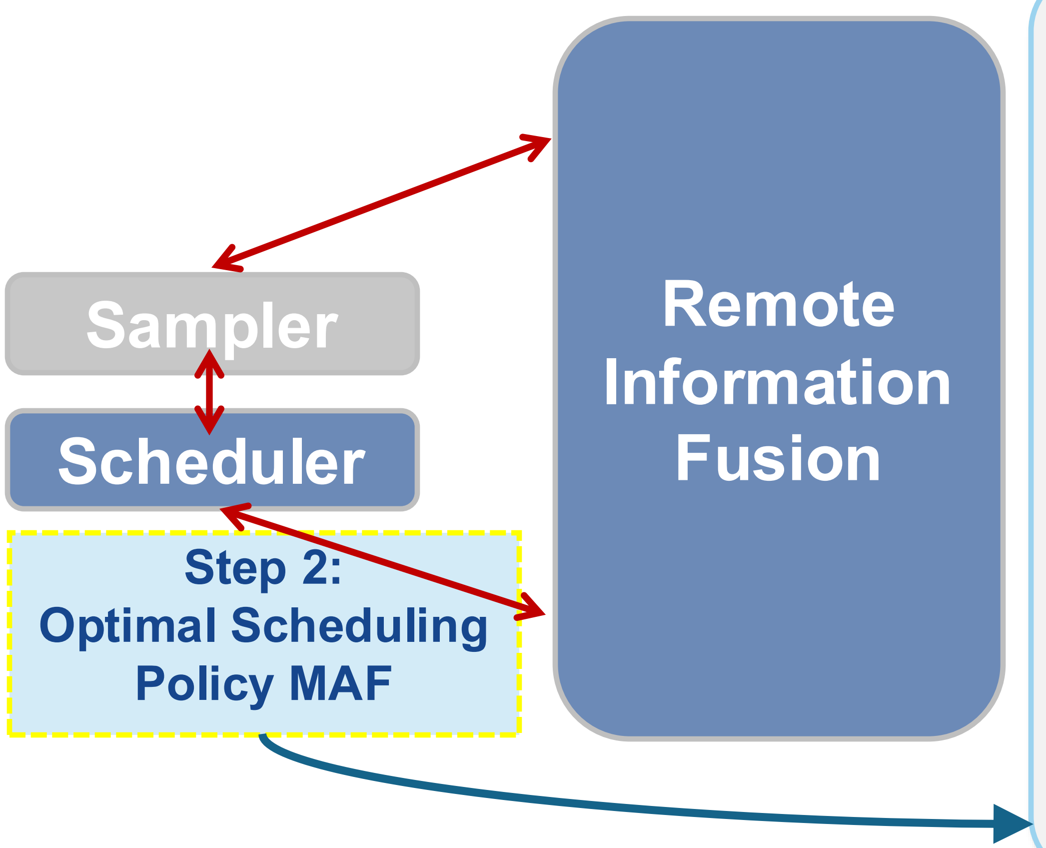
The **MMSE** g_{MMSE} is the optimal fusion policy:

$$\widehat{W}_t^{(1)} = \begin{cases} W_{t-\Delta_{1,t}} & \Delta_{1,t} \le \Delta_{2,t} \\ \underbrace{[G_{t,\rho}(\Delta_{1,t},\Delta_{2,t}) \quad Q_{t,\rho}(\Delta_{1,t},\Delta_{2,t})]}_{\text{Weighted Coefficients}} \cdot \underbrace{\begin{bmatrix} W_{t-\Delta_{1,t}} \\ W_{t-\Delta_{2,t}} \end{bmatrix}}_{\text{Weighted Sum}} & \Delta_{1,t} > \Delta_{2,t} \end{cases}$$

$$\widehat{W}_t^{(2)} = \begin{cases} W_{t-\Delta_{2,t}} & \Delta_{2,t} \le \Delta_{1,t} \\ \underbrace{[G_{t,\rho}(\Delta_{2,t},\Delta_{1,t}) \quad Q_{t,\rho}(\Delta_{2,t},\Delta_{1,t})]}_{\text{Weighted Coefficients}} \cdot \underbrace{\begin{bmatrix} W_{t-\Delta_{2,t}} \\ W_{t-\Delta_{1,t}} \end{bmatrix}}_{\text{Weighted Sum}} & \Delta_{1,t} > \Delta_{2,t} \end{cases}$$

Here, $\Delta_{i,t}$ is the age of source i.

Main Results: Optimal Scheduling



Decoupled Scheduling Policy

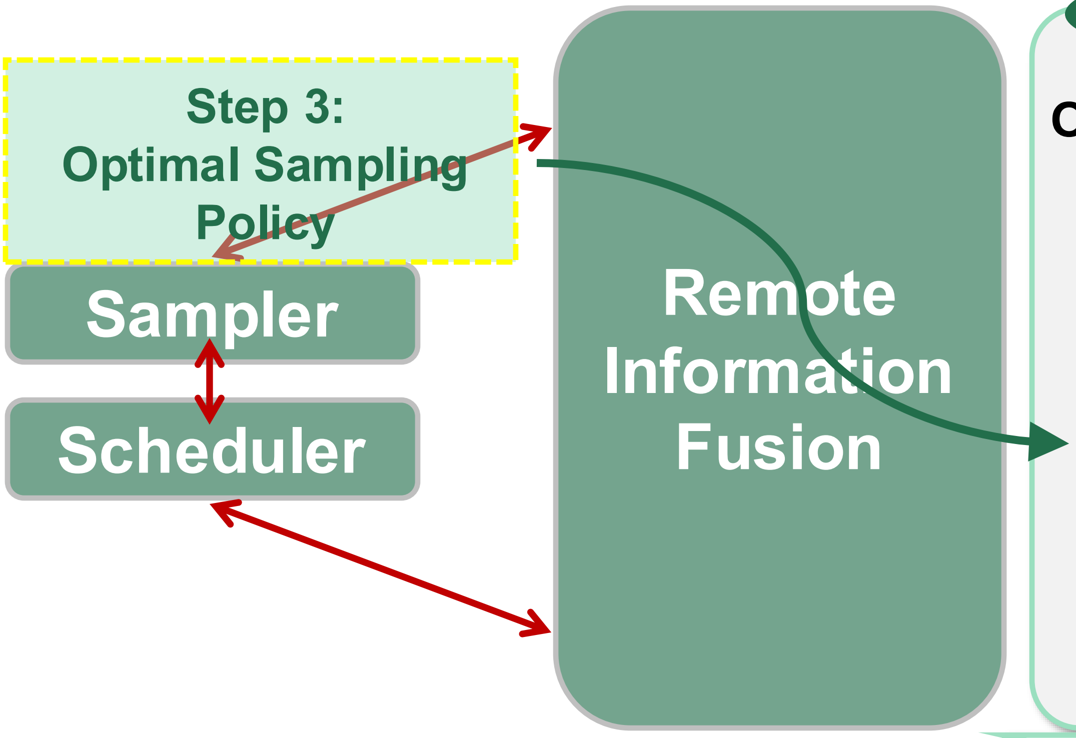
Optimal Scheduling Policy given g_{MMSE} and f (Theorem 2):
The Maximum Age First (MAF) π_{MAF} scheduling is optimal

$$\widehat{W}_t^{(1)} = \begin{cases} W_{t-\Delta_{1,t}} & \Delta_{1,t} \le \Delta_{2,t} \\ [G_{t,\rho}(\Delta_{1,t},\Delta_{2,t}) \quad Q_{t,\rho}(\Delta_{1,t},\Delta_{2,t})] \cdot \underbrace{\begin{bmatrix} W_{t-\Delta_{1,t}} \\ W_{t-\Delta_{2,t}} \end{bmatrix}}_{\textbf{Weighted Sum}} & \Delta_{1,t} > \Delta_{2,t} \end{cases}$$

$$\widehat{W}_t^{(2)} = \begin{cases} W_{t-\Delta_{2,t}} & \Delta_{2,t} \le \Delta_{1,t} \\ [G_{t,\rho}(\Delta_{2,t},\Delta_{1,t}) \quad Q_{t,\rho}(\Delta_{2,t},\Delta_{1,t})] \cdot \underbrace{\begin{bmatrix} W_{t-\Delta_{2,t}} \\ W_{t-\Delta_{1,t}} \end{bmatrix}}_{\textbf{Weighted Sum}} & \Delta_{1,t} > \Delta_{2,t} \end{cases}$$

Here, $\Delta_{i,t}$ is the age of source i.

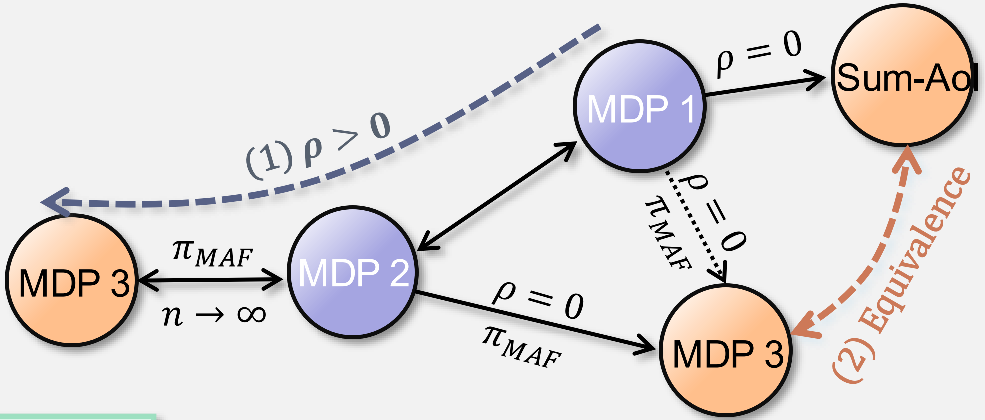
Main Results: Optimal Sampling



Optimal Sampling Policy

Optimal Sampling Policy given g_{MMSE} and π_{MAF} (Corollary 1):

Age-optimal Sampling[4] is All you Need!



$$\underbrace{\inf_{f} \limsup_{T\to\infty} \frac{1}{T} \mathbb{E}_{g_{MMSE},\pi_{MAF},f}\left[\int_0^T (W_t^{(1)} - \widehat{W}_t^{(1)})^2 + (W_t^{(2)} - \widehat{W}_t^{(2)})^2 dt\right]}_{\textbf{MSE-Oriented}} = \underbrace{\inf_{f} \limsup_{T\to\infty} \frac{1}{T} \mathbb{E}_{g_{MMSE},\pi_{MAF},f}\left[\int_0^T \Delta_{1,t} + \Delta_{2,t}\, dt\right]}_{\textbf{Sum-AoI-Oriented}}$$

[4] Ahmed M. Bedewy, Yin Sun, Sastry Kompella, and Ness B. Shroff. Optimal Sampling and Scheduling for Timely Status Updates in Multi-source Networks. IEEE Transactions on Information Theory, 2021

IV Numerical Results

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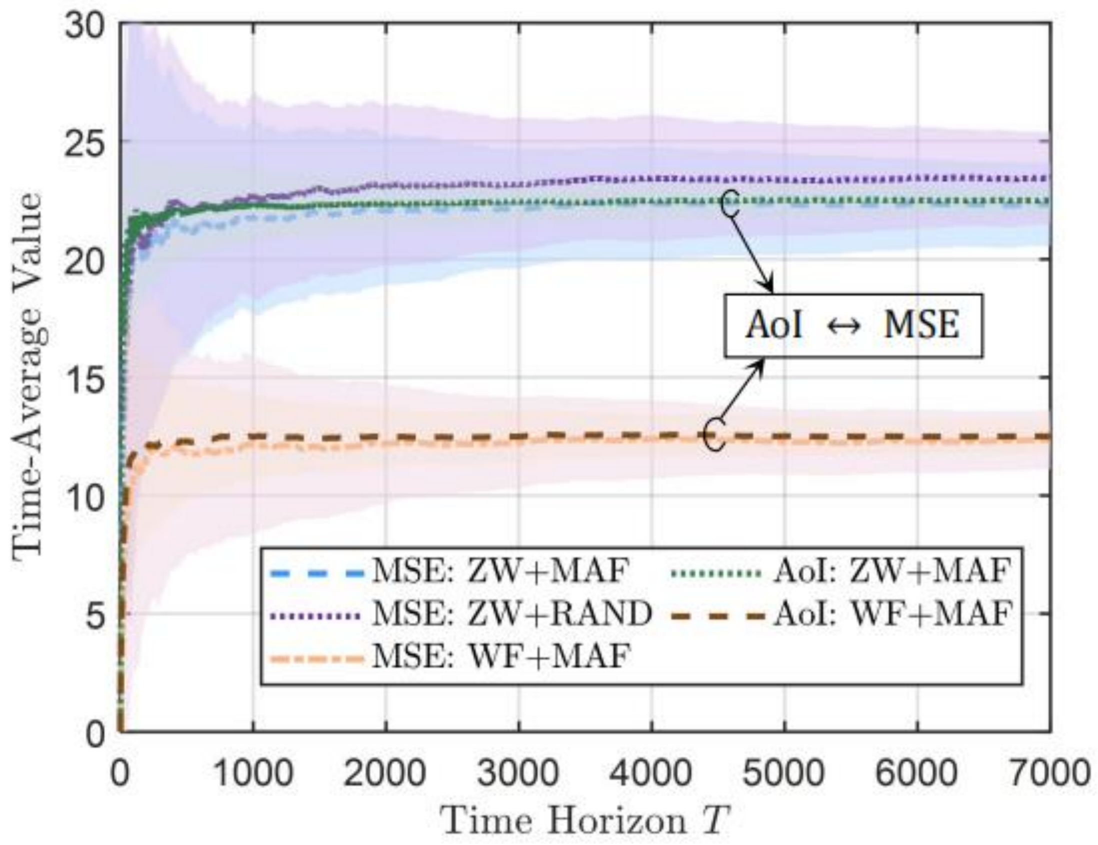
Simulations Results

Sum-AoI ↔ MSE



Insight:

- **Even for correlated sources, minimizing sum-AoI is equivalent to minimizing MSE.**

- **As a result, AoI can serve as a surrogate for remote estimation and information fusion for Wiener Processes, no matter whether the sources are correlated.**

Comparative Analysis: New Trends and Challenges

System Model in [5]

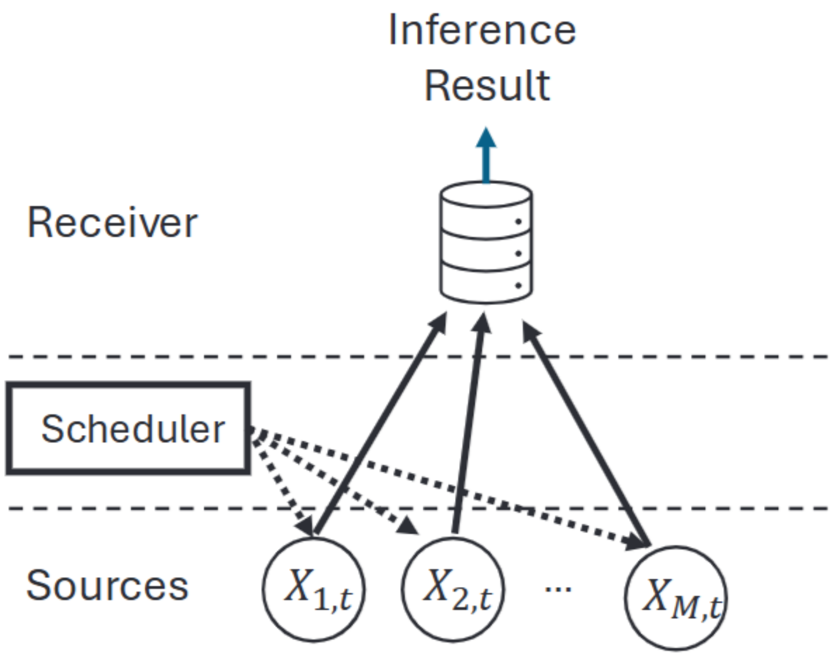


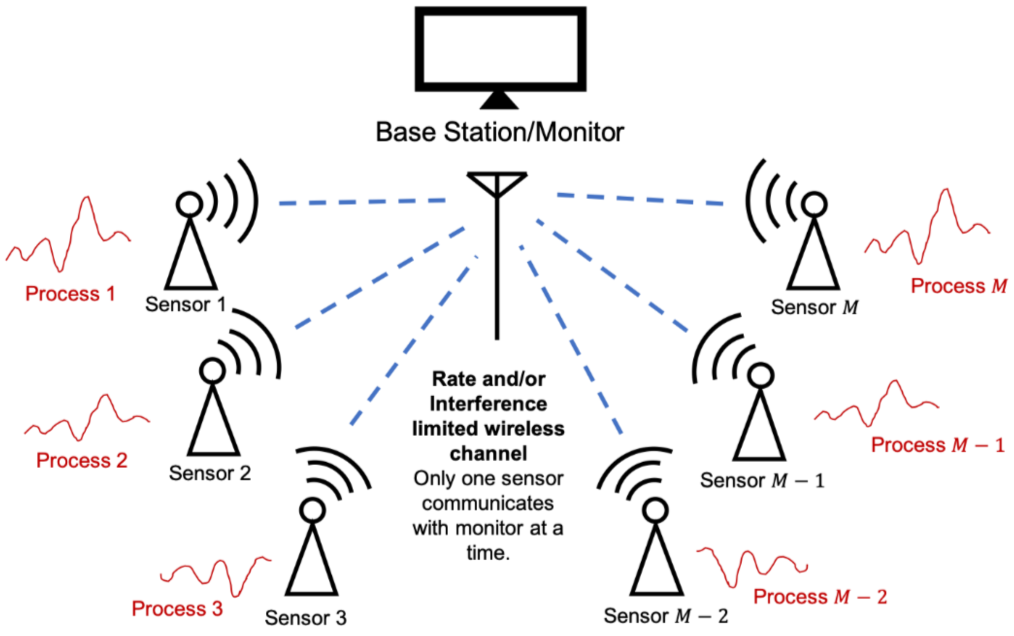
Fig. 1: A remote inference system with M correlated sources.

Comparsion	This Work	Arxiv Paper [5]
Objective	MSE (Not only function Age)	**A General Joint Function of Ages** (Age-based Inference Error)
Policies Components	**Sampling** & Scheduling & Fusion over **Continuous Time**	**Scheduling** over **Discrete Time Slots**
Channel	**Random Delay**	**Zero Delay**
Addressed Challenges	**Explicit Optimal Co-Design**	How to **Decouple** Ages for scheduling?

[5] Md Kamran Chowdhury Shisher, Vishrant Tripathi, Mung Chiang, and Christopher G. Brinton. AoI-based Scheduling of Correlated Sources for Timely Inference. Arxiv, **September 2, 2025.**

Comparative Analysis: New Trends and Challenges

System Model in [6]



Comparsion	This Work	TMC [6]
Objective	MSE	MSE
Policy Components	**Sampling** & Scheduling & Fusion over **Continuous Time**	**Scheduling Only** over **Discrete Time Slot**
Channel	**Random Delay**	**Zero Delay**
Addressed Challenges	**Explicit Optimal Co-Design**	How to Schedule **Correlated** Sources?
Sources	Non-stationary Markov	Non-stationary **Hidden** Markov

[6] R. V. Ramakanth, V. Tripathi and E. Modiano. Monitoring Correlated Sources: AoI-based Scheduling is Nearly Optimal, IEEE Transaction on Mobile Computing, **February, 2025**

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Thank you

On behalf of METU CNG

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